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PROBLEMS OF THE INTERACTION OF A BLUNT BODY WITH AN ACOUSTIC MEDIUM*

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The initial (supersonic) stage of the interaction of a blunt body (penetration and impact) with an acoustic medium (a compressible fluid) is examined in a laminar formulation. It is assumed that the boundary of the domain of interaction of the body with a medium moves at a velocity exceeding the velocity of sound in the medium. Explicit formulas are derived for the velocity of the particles of the medium and the pressure at each point of the interaction domain boundary. It is shown that the general solution of the linearized problem for the supersonic stage of blunt body penetration, given by an explicit formula /1-3/ in the form of a double integral, can be converted in such a manner as to reduce the formula to a single integral for an arbitrary body penetrating the fluid at an arbitrary velocity. Earlier only problems of the penetration of bodies of revolution bounded by second-order surfaces (cone /3, 4/, paraboloid /4, 5/, ellipsoid and hyperboloid /4/) at a constant velocity were investigated analytically using such a reduction. An exact expression is obtained for the law of motion on the inertia of a body of arbitrary shape after its contact with the fluid.

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1. Formulation of the problem. Let a rigid blunt body penetrate at a velocity $V(t)$ a weightless acoustic medium occupying the halfspace $z \geqslant 0$. It is assumed that the angle between the tangent plane to the body at points entering into interaction with the medium and the plane $z = 0$ is small in the whole time interval T under consideration. The boundary conditions from the body surface are carried over to the $z = 0$ plane. It is assumed that $V(t) \leqslant a$, $t \in [0, T]$, where a is the velocity of sound in the medium.

We take the origin of the Cartesian coordinate system at the point of initial tangency of the body with the medium. The z axis is directed into the depth of the medium while the x and y axes are along the initially free surface.

The motion of the particles of the medium is considered to be potential, i.e., the velocity $\mathbf{v}$ of the particles of the medium and the pressure p are defined in terms of the potential $\Phi(\mathbf{x}, t)$ by means of the formulas /1-3/

$$\mathbf{v}(\mathbf{x}, t) = \operatorname{grad} \Phi(\mathbf{x}, t), \quad p(\mathbf{x}, t) = -\rho \partial \Phi(\mathbf{x}, t)/\partial t, \quad x \in R_+^3 \tag{1.1}$$

where ρ is the density of the medium, and $\mathbf{x} = (x, y, z)$ is the vector of points of the half-space.

The potential Φ is a generalized, piecewise-smooth solution of the wave equation in R^4, i.e., it is continuous everywhere and smooth everywhere with the exception of the regular hypersurfaces S_k partitioning R^4 into a finite number of subdomains. The partial derivatives of Φ from each of the sides of the singular surface S_k have a unique one-sided limit. Outside of S_k the potential Φ satisfies the equation

$$\square \Phi(\mathbf{x}, t) = 0, \quad \mathbf{x} \in R_+^3, \quad t \in (0, T]; \quad \square \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{a^2}\frac{\partial^2}{\partial t^2} \tag{1.2}$$

while the conditions of a regular strong discontinuity (in the Sobolev sense) /6-8/ are satisfied on S_k

$$[\langle \mathbf{L}, \boldsymbol{\nu} \rangle] = 0; \quad \mathbf{L} \equiv \left\{ \frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y}, \frac{\partial \Phi}{\partial z} - \frac{1}{a^2}\frac{\partial \Phi}{\partial t} \right\} \tag{1.3}$$

$$[\mathbf{L}_4 a^2 \cos(\boldsymbol{\nu}, x_i) + L_i \cos(\boldsymbol{\nu}, t)] = 0, \quad i = 1, 2, 3; \quad x_1 = x,$$
$$x_2 = y, \; x_3 = z$$

Hence and henceforth, the square brackets denote the jumps in the quantities on passing through S_k while the angle brackets denote the scalar product, and ν is the unit vector normal to S_k in R^4.

We define the perturbation domain Ω in R^4 as the set of points to which the perturbation caused by submersion of the body would reach. Then

$$\Phi(\mathbf{x}, t) \equiv 0, \quad (\mathbf{x}, t) \in R^4 \setminus \Omega \tag{1.4}$$

since the medium is at rest at the initial instant.

The condition

$$\partial \Phi(x, y, 0, t)/\partial z = V(t), \quad (x, y) \in G(t), \; t \in (0, T] \tag{1.5}$$

is satisfied at points of opening of the domain of body contact with the medium G in the $z = 0$ plane.

For $t \in (0, T]$ the submersion is supersonic in nature if the inequality

$$\min_{(x, y) \in \partial G} \alpha(x, y, t) > a, \quad 0 < t \leqslant T \tag{1.6}$$

is satisfied.

The velocity α of motion of the boundary ∂G of the contact domain at the point $(x_0, y_0, 0)$ at the time t_0 equals

$$\alpha_0 = V_0 \, |\operatorname{grad} f(x_0, y_0)|^{-1}; \quad \alpha_0 = \alpha(x_0, y_0, t_0), \quad V_0 = V(t_0) \tag{1.7}$$

where f is the function governing the formula of the body. The boundary ∂G_0 of the domain G_0 $(G_0 = G(t_0))$ is determined by the equation

$$f(x, y) = H(t_0), \quad (x, y) \in \partial G_0; \quad H(t) \equiv \int_0^t V(\xi)\, d\xi \tag{1.8}$$

It is assumed in the penetration problem that $V(t)$ is a known function of the time. In the impact problem $V(t)$ is found from the equation of motion of the body in inertia

$$m \frac{dV}{dt} = -P(t), \quad P(t) = \iint_{G(t)} p(x, y, 0, t)\, dx\, dy \tag{1.9}$$

where m is the mass of the body and P is the force of its interaction with the medium.

Remarks. 1°. It follows from conditions (1.1) and (1.3) that the jump in the velocity vector is directed along the normal to the surface of discontinuity S_k while its magnitude is proportional to the jump in the pressure /7/

$$|[\mathbf{v}]| = [p]/(\rho a) \tag{1.10}$$

2°. The surfaces of strong discontinuity S_k are characteristic for the wave equation /6, 8/.

3°. The boundary of the contact domain can absorb energy. This energy is interpreted /8/ as the energy entrained by spray jets that occur during penetration /9/.

2. The pressure and velocity on the boundary of the contact domain. Let the point be $(x_0, y_0, 0) \in \partial G_0$. Then the limit values of the pressure p and the velocity vector $\mathbf{v}$ can be determined as equal (in the problems under consideration) to the jumps in the appropriate quantities during the approach to this point from the side of the contact domain.

The magnitudes of the velocity and pressure jumps at points of the contact domain boundary are determined by the expressions

$$|[\mathbf{v}]| = V_0\alpha_0 (\alpha_0^2 - a^2)^{-1/2}, \quad [p] = a\rho V_0\alpha_0 (\alpha_0^2 - a^2)^{-1/2} \tag{2.1}$$

Indeed, the boundary $S_0 = S(t_0)$ of a section through the domain Ω by the plane $t = t_0$ propagates at the velocity a (the velocity is measured along the normal to S_0). The cosine of the slope θ to the plane $z = 0$ is defined by the expression $\cos\theta = (\alpha_0^2 - a^2)^{1/2}/\alpha_0$. The vector $[\mathbf{v}]$ is directed along the normal to S_0 and its projection on the z axis at the point $(x_0, y_0, 0)$ equals V_0. Hence, taking account of the equality $|[v]| = V_0/\cos\theta$ and (1.10), we obtain (2.1).

3. The explicit form of the solution of the penetration problem in the supersonic case. It is known (for instance, /1-4, 10, 11/) that the solution of problem (1.2)-(1.5) has the form of a retarded potential

$$\Phi_0 = \Phi(\mathbf{x}_0, t_0) = -\frac{1}{2\pi}\iint \left\{\frac{\partial\Phi}{\partial z}\left(x, y, 0, t_0 - \frac{R_1}{a}\right)\right\}\frac{dx\,dy}{R_1} \tag{3.1}$$

$$R_1 = [(x - x_0)^2 + (y - y_0)^2 + z_0^2]^{1/2}$$

Formula (3.1) yields an explicit solution of the problem since the function in the braces is known over the whole $z = 0$ plane: if $(x, y) \in G(t_0 - R_1/a)$ then it equals $V(t_0 - R_1/a)$, otherwise it equals zero (see conditions (1.4) and (1.5)). However, the solution of specific problems by utilizing (3.1) is quite awkward (see /4/). We will later simplify (3.1) by converting the double integral into a single one.

Let us introduce the "lagging" time τ, $\tau > 0$. We note points on the surface $z = 0$ whose perturbation at the time τ affects the motion of the point $\mathbf{x}_0$ at the time t_0. The points noted generate a circle O_τ with centre at the point (x_0, y_0) of radius

$$l(\tau) = [a^2(t_0 - \tau)^2 - z_0^2]^{1/2} \tag{3.2}$$

Let $U(\tau) = O_\tau \cap G(\tau)$, $\varphi(\tau)$ be the angular measure of the set $U(\tau)$ and μ the set of points τ for which $\varphi(\tau) > 0$.

We examine all those points in the $z = 0$ plane whose perturbation in the time interval between τ and $\tau + d\tau$ exert an influence on the behaviour of the medium at the point $\mathbf{x}_0$ at the time t_0. The area of the set of these points is determined by the formula (we take (3.2) into account)

$$ds(\tau) = |l(\tau)\varphi(\tau)dl(\tau)| = a^2(t_0 - \tau)\varphi(\tau)d\tau \tag{3.3}$$

We have

$$R_1 = a(t_0 - \tau), \quad dxdy\,|_{(x, y)\in G(\tau)} = ds(\tau)$$

Substituting these expressions into (3.1) and taking account of (1.5) and (3.3), we obtain

$$\Phi_0 = -\frac{a}{2\pi}\int_\mu V(\tau)\varphi(\tau)\,d\tau \tag{3.4}$$

Formula (3.4) for the lagging potential is especially simple in the axisymmetric case.

4. Supersonic penetration of bodies of revolution in a fluid. In this case the domain $G(t)$ is a circle of variable radius $r_*(t)$.

4.1. Selfsimilar problems. It is known that if a body whose surface shape is described by a positive, smooth, homogeneous function of degree d penetrates an acoustic medium at a velocity $V(t) = V(1)t^{d-1}$, $d \geqslant 1$, then the problem is selfsimilar

$$\Phi(\mathbf{x}, t) = (t/t_1)^d\Phi(t_1\mathbf{x}/t, t_1)$$

The domain $G(t)$ is obtained from the domain $G(t_1)$ by conversion of the homothety, where the velocity of domain boundary motion is constant and $r_*(t) = Mat$, and $M \equiv \alpha(x, y, t)/a = \text{const}$.

We will examine the problem of determining the set μ in (3.4).

Different cases of the intersection of the circle O_τ with the domain $G(\tau)$ are possible depending on the values of the quantities r_0, z_0 and M.

Case 1. For $\tau = 0$ the centre of the domain $G(\tau)$ lies outside a circle of radius $l_0 = l(0)$ with centre at a point with coordinates $(r_0, 0)$, $r_0 = (x_0^2 + y_0^2)$, i.e., $r_0^2 + z_0^2 > a^2t^2$. Then the following modifications are possible.

1a. The boundary of the circle $G(\tau)$ does not intersect the circle O_τ. Then the wave will not arrive at the point $\mathbf{x}_0$ and the velocity $\mathbf{v}(\mathbf{x}_0, t_0)$ equals zero.

1b. The circle $G(\tau)$ grows, touches the circle O_τ at the time τ_- and intersects it up to the time τ_+ and then recedes from it. Then the set μ is the interval (τ_-, τ_+). From the cosine theorem we then have

$$\varphi = 2 \arccos \frac{r_0^2 + l^2(\tau) - r_*^2(\tau)}{2r_0 l(\tau)}, \quad r_0 - r_*(\tau_\mp) = l(\tau_\mp) \tag{4.1}$$

1c. The circle $G(\tau)$ grows and starts to intersect the circle O_τ at the time τ_- while it encloses it completely **from the time** τ_+. We then obtain

$$\mu \equiv (\tau_-, \tau_k), \quad \tau_k = t_0 - z_0/a \tag{4.2}$$

$$\varphi = \begin{cases} 2 \arccos \dfrac{r_0^2 + l^2(\tau) - r_*^2(\tau)}{2r_0 l(\tau)}, & \tau_- < \tau \leqslant \tau_+ \\ 2\pi, & \tau_+ \leqslant \tau < \tau_k \end{cases}$$

We find the quantities τ_- and τ_+ from the conditions

$$r_0 - r_*(\tau_\mp) = \pm l(\tau_\mp), \quad r_0 > l_0 \tag{4.3}$$

Case 2. For $\tau = 0$ the centre of the domain $G(\tau)$ lies inside the circle O_τ of radius l_0. Then the following modifications are possible.

2a. The circle $G(\tau)$ grows, is tangent to the circle O_τ at the time τ_- and intersects it to the time τ_+ and then recedes from it. Then the angle $\varphi(\tau)$ is determined from the first formula in (4.1) while the set μ is the interval (τ_-, τ_+) where

$$r_0 \pm r_*(\tau_\mp) = l(\tau_\mp) \tag{4.4}$$

2b. The circle $G(\tau)$ grows, starts to intersect the circle O_τ at the time τ_- and encloses it completely from the time τ_+. Then the set μ and the angle $\varphi(\tau)$ are determined by (4.2), the quantity τ_- by (4.4) with the upper signs, and τ_+ by (4.3) with the lower signs.

Solving (4.1) and (4.3), we obtain

$$\tau_\mp = (Mr_0 - at_0 \mp \sqrt{D})/[a(M^2 - 1)], \quad r_0 > l_0 \tag{4.5}$$

$$D = (Mr_0 - at_0)^2 - (M^2 - 1)(r_0^2 + z_0^2 - a^2t_0^2)$$

For $r_0 < l_0$ the expression for τ_+ corresponds to (4.5) and for τ_- it corresponds to (4.5) with the formal replacement of a by $-a$.

In the case $r_0 > l_0$ if $D < 0$ then modification 1a is realized, if $D \geqslant 0$ and $\tau_+ < \tau_* = r_0/(Ma)$, then modification 1b, if $D \geqslant 0$ and $\tau_+ > \tau_*$ then modification 1c.

In the case when $r_0 < l_0$, if $\tau_+ < \tau_*$, modification 2a is realized; if $\tau_+ > \tau_*$ modification 2b is realized.

4.2. Determination of the velocity potential in the $z = 0$ plane. Because of the arbitrariness of the function f and the conditions of the problem, $r_*(\tau)$ can be an arbitrary function such that

$$r_*^{\cdot}(\tau) > a \tag{4.6}$$

Different cases of the intersection of O_τ with $G(\tau)$ are possible depending on the values of r_0 and the form of the law of variation of $r_*(\tau)$.

1) $r_0 > l_0 = at_0$. In this case modifications 1a and 1c are possible and (4.2) and (4.3) will hold. Modification 1b is not realized becuase of condition (4.6).

2) $r_0 < l_0$. In this case only modification 2b is possible and (4.2) and (4.3) with the lower signs and (4.4) with the upper signs will be valid. Modification 2a is not possible because of condition (4.6).

Substituting the expression for φ from (4.2) and the dependences (4.3) and (4.4) into (3.4), we obtain

$$\Phi(r_0, 0, t_0) = -a\left\{\frac{1}{\pi}\int_{\tau_-}^{\tau_+} V(\tau) \arccos \frac{K_+}{L} d\tau + \int_{\tau_+}^{t_0} V(\tau)\, d\tau\right\} \tag{4.7}$$

$$K_{\pm} = r_0^2 \pm a^2 (t_0 - \tau)^2 - r_*^2 (\tau), \quad L = 2r_0 a (t_0 - \tau)$$

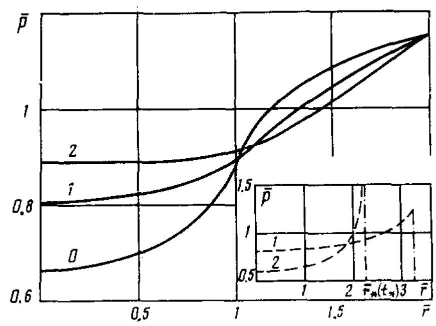
Taking account of (1.1), we find the pressure distribution in the domain $G(t_0)$ from (4.7)

$$p(r_0, 0, t_0) = a\rho \left\{ \frac{1}{\pi} \int_{\tau_-}^{\tau_+} V(\tau) \frac{K_-}{(t_0 - \tau)\sqrt{L^2 - K_+^2}} d\tau + V(t_0) \right\} \tag{4.8}$$

Remarks. 1°. For $r_0 = 0$ we obtain $\tau_- = \tau_+$ from (4.4), and then from (4.7) we have

$$\Phi(0, 0, t_0) = -a \int_{\tau_+}^{t_0} V(\tau)\, d\tau, \quad p(0, 0, t_0) = a\rho \left\{ V(t_0) - V(\tau_2) \frac{a}{a + r_*^{\cdot}(\tau_+)} \right\} \tag{4.9}$$

The integrand in (4.8) has a singularity at $r_0 = 0$; consequently, it is best to use (4.9) for a numerical computation.



2°. The formula obtained earlier /12/ by integral transform methods for the pressure, analogous to (4.8), is given incorrectly in /12/.

As an example the figure shows as continuous lines the dependence of $\bar{p} = p(r_0, 0, t_0)/(\rho a V(t_0))$ on $\bar{r} = r_0/(at_0)$ in selfsimilar problems for $M = 2$; the k-th curve corresponds to the degree of selfsimilarity of the problem $d = 1 + k/2$ $(k = 0, 1, 2)$. In these cases $\tau_- = (r_0/a - t_0)/(M - 1)$ for $r_0 > l_0$ and $\tau_- = (t_0 - r_0/a)/(M + 1)$ for $r_0 < l_0$; $\tau_+ = (t_0 + r_0/a)/(M + 1)$.

Note that for $r_0/(at_0) = M$ all the curves converge at one point $M/\sqrt{M^2 - 1}$. This result is a consequence of (2.1). In the special case when $d = 1$ it was obtained earlier /3/.

4.3. Determination of the potential and pressure on the z axis. The circle O_τ constructed for points on the z axis touches the boundary of the circle $G(\tau)$ simultaneously at all its points at the time τ_-, and then is completely enclosed by it. Then the set μ is the interval $(\tau_-, t_0 - z_0/a)$, and $\varphi \equiv 2\pi$. We obtain from (3.4)

$$\Phi(0, z_0, t_0) = -a \int_{\tau_-}^{t_0 - z_0/a} V(\tau)\, d\tau \tag{4.10}$$

We find the quantity τ_- from the condition $r_*(\tau_-) = l(\tau_-)$ or taking account of (1.8) and (3.2), we have $f(\sqrt{a^2 (t_0 - \tau_-)^2 - z_0^2}) = H(\tau_-)$. We obtain from (1.1) and (4.10)

$$p(0, z_0, t_0) = a\rho \{V(t_0 - z_0/a) - V(\tau_-)\partial\tau_-/\partial t_0\}$$

$$\frac{\partial \tau_-}{\partial t_0} = \left\{ 1 - M(\tau_-) \left[1 - \left(\frac{z_0}{a(t_0 - \tau_-)} \right)^2 \right]^{1/2} \right\}^{-1}, \quad M(\tau_-) = \alpha(r_*, \tau_-)$$

The potential along the axis of revolution was determined earlier /4/ in an analogous problem (for a constant penetration velocity) but errors were admitted here in the computations.

5. The impact of three-dimensional bodies on an acoustic medium. It is known /13/ that the force acting on a body of arbitrary shape that penetrates into an acoustic half-space at an arbitrary velocity $V(t)$ equals

$$P(t) = P_1(t) + P_2(t), \quad P_1(t) = \rho a V(t) Q_0(t), \quad P_2(t) = \rho a V_1(t) Q_1(t) \tag{5.1}$$

where Q_0 is the area of the domain $G(t)$, Q_1 is the area of that domain outside $G(t)$ whose particles are involved in the motion, and V_1 is the mean particle velocity on the area Q_1.

The area is $Q_1 = 0$ for supersonic motion of the points $\partial G(t)$. If $Q_1 \neq 0$, then $V_1 < 0$ /1/ under the ordinary conditions of body impact on a fluid. Therefore, $P_1(t) \geqslant P(t)$, where the equality is conserved until $\alpha(x, y, t) > a$.

For $\alpha(x, y, t) > a$ we have from (1.9) and (5.1) (taking into account that $V^{\cdot} = V dv/dH$, $v[H(t)] \equiv V(t)$)

$$m\, dv/dH = -\rho a Q_0(H)$$

Integrating this equation, we obtain an expression for the body velocity and a relationship between the time t and the depth of submersion H

$$v(H)=V_0-\rho a m^{-1}F(H);\quad F(H)=\int_0^H Q_0(h)\,dh,\quad V_0=V(0)=v(0) \tag{5.2}$$

$$t=\int_0^H \frac{dh}{V_0-\rho a m^{-1}F(h)}$$

Here $F(H)$ is the body volume under a cut at the height H.

Formulas (5.2) yield the law of motion in inertia for a blunt body of arbitrary shape after its impact at a velocity V_0 on the surface of an acoustic half-space.

Let us find the depth H_* at which the supersonic nature of the motion of the domain boundary is lost. We obtain from (1.6)-(1.8) and the first formula in (5.2)

$$\frac{V_0}{g(H_*)}=a\left\{1+\frac{\rho}{m}\frac{F(H_*)}{g(H_*)}\right\},\quad g=\max_{\mathbf{x}\in\partial G(H)}|\operatorname{grad} f| \tag{5.3}$$

In particular, if the body is an elliptical paraboloid, i.e., $f(x, y)=Ax^2+By^2,\quad B>A>0$, we have

$$Q_0(h)=\frac{\pi h}{\sqrt{AB}},\quad F(H)=\int_0^H Q_0(h)\,dh=\frac{\pi H^2}{2\sqrt{AB}}$$

Substituting this expression into (5.2) and integrating, we obtain

$$t=\beta^{-1}\ln\left|\frac{1+H\beta/(2V_0)}{1-H\beta/(2V_0)}\right|,\quad \beta=\left(\frac{2\pi\rho a V_0}{m\sqrt{AB}}\right)^{1/2} \tag{5.4}$$

We find the expressions

$$H(t)=2V_0\beta^{-1}(e^{\beta t}-1)(e^{\beta t}+1)^{-1} \tag{5.5}$$

$$V(t)=4V_0e^{\beta t}(e^{\beta t}+1)^{-2}$$

$$P(t)=-4V_0m\beta(e^{\beta t}-e^{2\beta t})(e^{\beta t}+1)^{-3}$$

from (5.4), that yield the exact solution of the problem of the impact of an elliptical paraboloid to the surface of an acoustic half-space.

Taking account of (1.6) and (5.3), we determine the time of supersonic mode termination t_* $(\alpha(r, t_*)=a)$ from the equation

$$V(t_*)-2a\sqrt{H(t_*)B}=0$$

From (5.5) it can be obtained that $V^{\cdot\cdot}(t_m)=0$, where $t_m=\beta^{-1}\ln(2+\sqrt{3})$. Hence, we have the upper bound for the maximal fluid drag force

$$\max_t P(t)\leqslant P_1(t_m)=mV_0\beta\frac{20+12\sqrt{3}}{(3+\sqrt{3})^3}\approx 0{,}965\left(\frac{m\rho a V_0^3}{\sqrt{AB}}\right)^{1/2}$$

Note that expressions analogous to (5.5) have been obtained in /14/ in the problem of body impact on a surface of an isotropic elastic half-space.

We find an expression for the velocity potential in the $z=0$ plane from (4.7) and (5.5) for the case of impact by a paraboloid of revolution on an acoustic medium $(A=B)$

$$\Phi(r_0, 0, t_0)=-4V_0a\left\{\pi^{-1}\int_{\tau_-}^{\tau_+}\frac{e^{\beta\tau}}{(e^{\beta\tau}+1)^2}\arccos\frac{K_+}{L}\,d\tau+\right.$$

$$\left.\beta^{-1}\left[(e^{\beta\tau_+}+1)^{-1}-(e^{\beta t_0}+1)^{-1}\right]\right\}$$

$$r_*(t)=\left(\frac{2V_0}{\beta A}\frac{e^{\beta t}-1}{e^{\beta t}+1}\right)^{1/2}$$

The quantities τ_- and τ_+ are determined from (4.3) and (4.4).

Graphs of the pressure distribution $\bar{p}=p(r_0, 0, t_0)/(\rho a V_0)$ over the interaction domain are represented by dashed lines in the figure for $t_0=t_*/2$ (curve *1*) and $t_0=t_*$ (curve *2*) for $\rho=1\ \text{g/cm}^3$, $a=1500$ m/sec, $A=0.1\ \text{m}^{-1}$, $V_0=100$ m/sec, and $m=100$ kg.

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THE PLANE STATE OF STRESS OF AN ELASTIC PLANE WITH TWO INTERSECTING SLITS*

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The construction of an exact closed solution for the problem of stress concentration in an elastic plane near two rectilinear slits of identical length that intersect at the centre at an arbitrary angle is proposed. An arbitrary rupturing and shearing load is applied along the slit edges. The construction of the solution of the problem is based on its reduction to a Riemann problem with a matrix coefficient of special structure that allows solution by quadratures. The possibility of solving such a problem was mentioned in /1/. This solution was first constructed for the case when the index $\varkappa_\varepsilon = 0$ for the ratio of the characteristic functions of the matrix coefficient /2/, and then also for $\varkappa_\varepsilon \neq 0$ /3/.

A different method from that described in /3/ is proposed for solving the Riemann problem for the cases when $\varkappa_\varepsilon = 1$ and $\varkappa_\varepsilon = -1$. The solution of the problem, constructed by quadratures, is converted to a form convenient for numerical realization. Computational formulas are obtained for the stress intensity factors.

1. Formulation of the problem of intersecting slits and its separation into an auxiliary problem. We investigate the plane state of stress of an elastic plane with two slits of identical length $2b$ (without loss of generality, we consider $b = 1$) that intersect at the centre at an arbitrary angle 2α (problem T). We take the bisectrix of the large angle between the slits as the horizontal axis of symmetry (Fig.1). As usual, the positive direction of variation of the angle θ is counter-clockwise. A positive load

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